AP Calculus Riemann Sums & Integration

Name: _____ **Date:** _____

Class Period: _____

Part I: Indefinite Integrals

Evaluate the following indefinite integrals. Don't forget the constant of integration.

1. $\int (4x^3 - 6x^2 + 5)\,dx$

2. $\int \left(\sqrt{x} + \frac{3}{\sqrt[3]{x}}\right)\,dx$

3. $\int \frac{x^2 + 4x - 1}{x^2}\,dx$

4. $\int (2\sin(x) + 3\cos(x))\,dx$

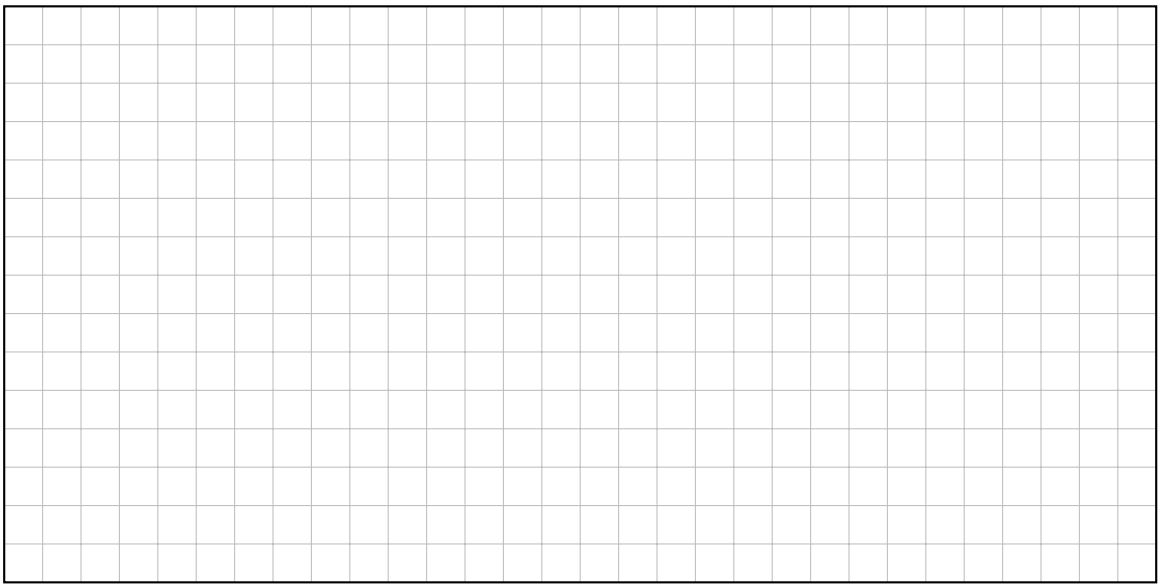
5. $\int (\sec(y)(\tan(y) - \sec(y))\,dy$

Part II: Riemann Sums

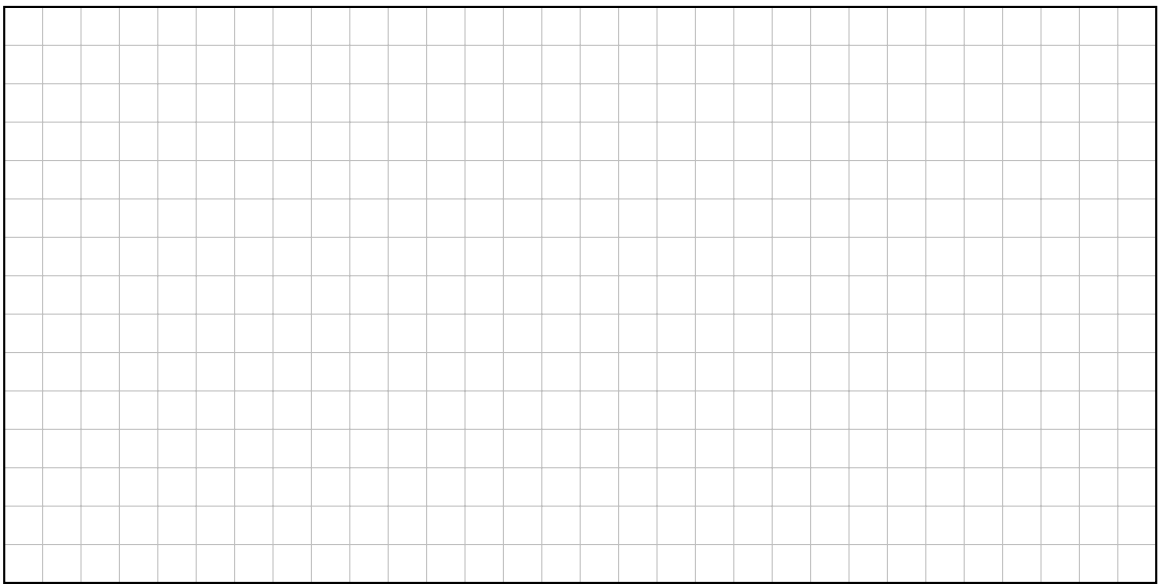
Use the data or functions provided to approximate the area under the curve.

6. The velocity of a particle, $v(t)$, is given in the table below. Use a **Left Riemann Sum** with 4 subintervals indicated by the data to approximate the total distance traveled, $\int_0^{10} v(t)\,dt$.

t (seconds)	0	2	5	9	10
$v(t)$ (ft/s)	4	6	8	5	2



7. Let $f(x) = x^2 + 1$. Estimate the area bounded by $f(x)$, the x-axis, $x = 0$, and $x = 4$ using a **Right Riemann Sum** with 4 equal subintervals.



Part III: Definite Integrals (The Fundamental Theorem of Calculus)

Evaluate the definite integrals exactly.

8. $\displaystyle\int_{-1}^{2} (3x^2 - 2x)\,dx$

9. $\displaystyle\int_{0}^{\pi} (2\sin x + \cos x)\,dx$

10. $\displaystyle\int_{1}^{4} \left(\frac{5}{x^3} - \sqrt{x}\right)\,dx$

11. $\displaystyle\int_{0}^{2} (x-1)^2\,dx$

Part IV: Average Value of a Function

Find the average value of the function on the given interval.

12. Find the average value of $f(x) = 3x^2 - 6x$ on the interval $[1, 3]$.

13. Find the average value of $g(x) = \sec^2 x$ on the interval $[0, \frac{\pi}{4}]$.

14. Find the value(s) of c guaranteed by the Mean Value Theorem for Integrals for the function $h(x) = 2x$ on the interval $[0, 4]$ such that $h(c) = h_{\text{avg}}$.
To do this, first find the average value. That average value must occur at $f(c)$. Set $f(c) =$ [average value], then solve for c.

Part V: The Second Fundamental Theorem of Calculus

Find the derivative of the defined functions.

15. Let $F(x) = \displaystyle\int_2^x \sqrt{t^3 + 1}\, dt$. Find $F'(x)$.

16. Let $g(x) = \displaystyle\int_1^{x^3} \cos(t^2)\, dt$. Find $g'(x)$.

17. Let $H(x) = \displaystyle\int_\pi^{\sqrt{x}} \frac{t}{t+1}\, dt$. Find $H'(x)$.